

## Through process stochastic model of hot strip rolling

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**Abstract.** Advanced numerical models, which predict heterogeneity of microstructural features, are needed to design modern multiphase steels. Models based on stochastic internal variables meet this requirement. Our objective was to account for the random character of the recrystallization and to transfer this randomness to equations describing the evolution of the dislocations and the grain size during hot deformation. The idea of the internal variable model with the dislocation density and the grain size being stochastic variables is described briefly in the paper. Histograms of the grain size measured in the experimental compression tests were used to identify the coefficients in the model. Inverse analysis with the objective function based on the distance between histograms was applied. The model was used to simulation of the various technological routes in the industrial process of the hot strip rolling.

### Introduction

Continuous development of the industry is associated with the search for new construction materials that combine high strength with good formability, as well as high strength-to-density ratio. These features can be obtained for steels with heterogeneous microstructures, in which hard constituents of bainite, martensite and retained austenite are dispersed in a soft ferrite matrix [1]. Advanced numerical models are needed to gain knowledge on the local heterogeneities of the microstructural features and resulting properties. As it was shown in [2] internal variable stochastic models, which can predict heterogeneity of the microstructure, would be a support for the design of multiphase materials.

Majority of the models, which can predict heterogeneity of the microstructure, is based on the explicit representation of the microstructure using Representative Volume Elements (RVE). These models require long computing times. Development of the fast mean field model with a capability to predict distributions of the selected microstructural features is the motivation for our research. The papers, which deal with an application of the mean-field stochastic approach to describe heterogeneity of the microstructure, are rare and they usually consider problem of a correlation between material microstructure and in-use properties of products. Among the recent papers, authors of [3] used statistics for geometry description in homogenization methods and in [4]  $n$ -point statistics was applied to predict crack probability. In few papers random distribution of selected parameter is used as an input for the deterministic material model, e.g. the Authors of [5] studied an influence of various distributions of the austenite grain size prior to recrystallization on the recrystallization kinetics. Our objective was to account for the random character of the recrystallization which occurs during deformation and after deformation at elevated temperatures and to transfer this randomness to equations describing evolution of the dislocation populations

and the grain size. The idea of the model is presented in [6] and its identification for hot deformation processes is described in [2,7]. Since the model incorporates stochastic character of the phenomena, it may predict the distribution of parameters instead of their average values. The general objectives of the present work were twofold. The first was identification and validation of the stochastic approach to modelling hot deformation of the dual phase (DP) steel including dynamic processes during deformation and static processes during interpass times. The second was application of this model to the simulation of various technological routes for the hot strip rolling process. Application of the results from the stochastic model as input for the simulation of phase transformations during laminar cooling recapitulates the paper.

### Model

Description of the model.

The model is discussed in details in [6] and only main equations are repeated here for a completeness of the presentation. The model originates from Kocks-Estrin-Mecking (KEM) approach [8,9] with the stochastic dynamic recrystallization term added. After discretization in time, the evolution of the dislocation density is governed by the equation:

$$\rho(t_i) = \rho(t_0)[1 - \xi(t_i)] + \left\{ \rho(t_{i-1}) + [A_1 \dot{\epsilon} - A_2 \rho(t_{i-1}) \dot{\epsilon}^{1-a_7}] \Delta t \right\} \xi(t_i) \quad (1)$$

where:  $t$  - time,  $\dot{\epsilon}$  - strain rate,  $a_7$  - coefficient accounting for the influence of the strain rate on the recovery,  $\Delta t$  - time step,  $A_1, A_2$  - coefficients responsible for hardening and recovery defined in [6] and in Fig. 1.

The parameter  $\xi(t_i)$ , accounts for a random character of the recrystallization and its distribution is described by the following conditions:

$$\mathbf{P}[\xi(t_i) = 0] = \begin{cases} p(t_i) & \text{if } p(t_i) < 1 \\ 1 & \text{otherwise} \end{cases} \quad (2)$$

$$\mathbf{P}[\xi(t_i) = 1] = 1 - \mathbf{P}[\xi(t_i) = 0]$$

In Eq. 2  $p(t_i)$  is a function, which bounds together the probability that the material point recrystallizes in a current time step and present state of the material:

$$p(t_i) = a_4 \rho(t_{i-1})^{a_6} \frac{3\gamma(t_i)\tau}{D_\gamma(t_{i-1})} \exp\left(\frac{-a_5}{RT}\right) \Delta t \quad (3)$$

where:  $D_\gamma$  - grain size in  $\mu\text{m}$ ,  $\tau$  - energy per unit dislocation length,  $a_4, a_5, a_6, a_{17}$  - coefficients.

In Eq. 3 variable  $\gamma$  represents a mobile fraction of the recrystallized grain boundary and depends on the distribution of  $\xi(t_{i-1})$  in previous step, as it is explained in [6]. In the work [7] the model was further extended by including interpass times and static phenomena into simulations. Several trajectories of Eq. 1 were calculated, each time using randomly generated values of  $\rho(t_0)$  and  $D_\gamma(t_0)$ . These individual trajectories were then aggregated into histograms at consecutive time steps  $t_i$ . Strictly speaking, we start with the grain size  $D_\gamma(t_0) \equiv D_0$  which is a random variable described by the Weibull distribution with the shape parameter  $k = 10$ , which was determined on the basis of measured grain size distributions shown in [2]. The scale parameter of the distribution was established as the average grain size measured after roughing rolling.

The changes of the grain size during deformation and during the interpass times were calculated based on the fundamental works of Sellars [10], who proposed equations describing grain size after static and dynamic recrystallization, as well as after grain growth.

When during the calculation the random parameter  $\xi(t_i) = 0$ , the considered point recrystallizes and its new grain size  $D_\gamma(t_i)$  is drawn from the Gauss distribution with the expected value of the grain size calculated by the deterministic model either for the static or for the dynamic recrystallization [10]. The standard deviation in the Gauss distribution is an optimization variable  $a_{16}$  in the model. The chart showing flow of calculations in the model is shown in Fig. 1, where:  $N_p$  - number of the Monte Carlo points,  $N_t$  - number of the time steps,  $\dot{\epsilon}$  - strain rate,  $b$  - module of the Burgers vector,  $l$  - mean free path for the dislocations,  $R$  - universal gas constant,  $T$  - temperature in K.

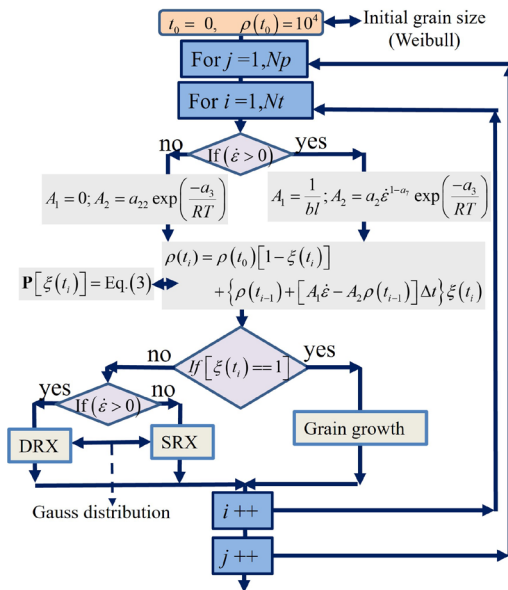


Fig. 1. Selected results of a comparison of measured and calculated flow stress in uniaxial compression.

The conventional objective function used in the identification of the deterministic models [11] was extended by including the metric of the distance between measured and calculated histograms of the grain size. The experimental data published in [12] were used. Steel DP600 with the symbol S406 in that publication was considered. The tests composed uniaxial compression of the samples measuring  $\phi 10 \times 12$  at various temperatures and strain rates. The total strain was 1 in all the tests and the expected value of the austenite grain size after preheating was  $30 \mu\text{m}$ . Full set of the experimental data for these test, including micrographs taken at various stages of the deformation and after the deformation, can be found in the RFCS report [13]. The model with optimal coefficients was validated and the selected results are shown in Fig. 2 for the flow stress and in Fig. 3 for the austenite grain size. The experimental flow stress was calculated by the inverse solution [11] for the forces measured in the uniaxial compression tests. Reasonably good agreement between measurements and predictions of the model was obtained for all the experiments.

Beyond the histograms of the selected parameters, the model allows calculation of the average dislocation density  $\rho_{av}$  and further the flow stress  $\sigma_p$ , as follows:

$$\sigma_p = \alpha G b \sqrt{\rho_{av}} \quad \rho_{av} = \frac{1}{N_p} \sum_{i=1}^{N_p} \rho_i \quad (4)$$

where:  $G$  - shear modulus,  $b$  - module of the Burgers vector,  $N_p$  - number of points in the Monte Carlo solution,  $\alpha$  - coefficient.

The model contains 22 coefficients, which are grouped in the vector  $\mathbf{a}$ . These coefficients were identified on the basis of the experimental data.

Identification and validation of the model.

Details of the identification procedure are described in [2,6]. Inverse algorithm developed in [11] was applied.

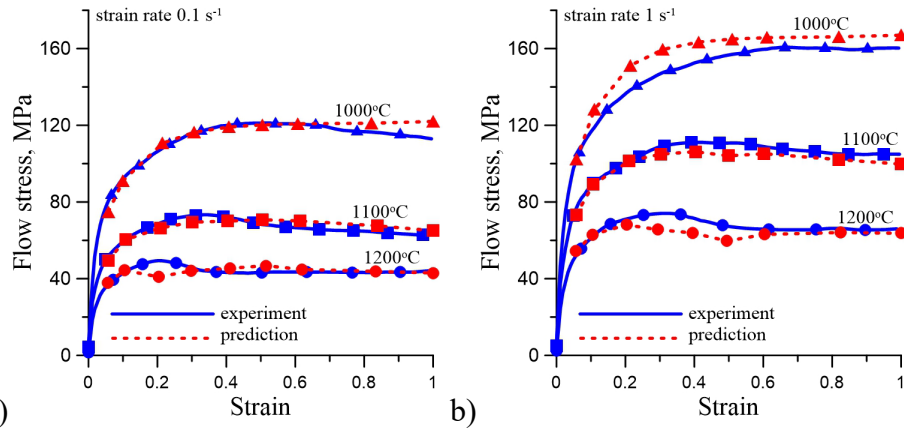


Fig. 2. Selected results of a comparison of measured and calculated flow stress in uniaxial compression.

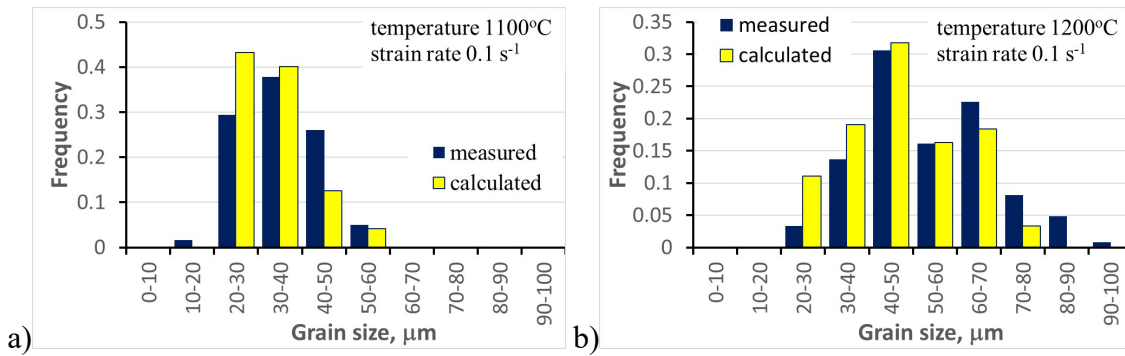


Fig. 3. Selected results of a comparison of measured and calculated histograms of the grain size after uniaxial compression at the strain rate  $0.1 \text{ s}^{-1}$  and temperature  $1100^\circ\text{C}$  (a) and  $1200^\circ\text{C}$  (b).

We assumed further that the model, which was validated in the laboratory experiments for various deformation/temperature cycles, will give good results for the industrial conditions.

### Numerical Tests of the Model

Numerical tests were performed to evaluate model’s predictive capability in the varying conditions of deformation. The stochastic model was implemented into the finite element (FE) program [14] and it was solved in selected points of the FE mesh. The two experiments were simulated: i) Interrupted compression of the uniaxial samples with various holding times between deformations, ii) Varying strain rate test.

Interrupted compression of the uniaxial samples [15].

This test is usually used to develop static recrystallization model. In the present work simulation of the 2 step compression of the sample measuring  $\phi 10 \times 12 \text{ mm}$  were performed. Results for the strain of 0.4 in both deformations and various interpass times are shown in Fig. 4 and in Fig. 5. The deformation temperature was  $1000^\circ\text{C}$  and the strain rate was  $1 \text{ s}^{-1}$ . Changes of the flow stress calculated following Eq (4) are shown in Fig. 4. The flow stress during the interpass time decreases asymptotically to zero but it can be concluded that the recrystallization is completed in 2 s after the end of deformation. Histograms of the grain size and dislocation density at the beginning of the second deformation for various interpass times are shown in Fig. 5. It is confirmed that after 2 s the static recrystallization is completed.

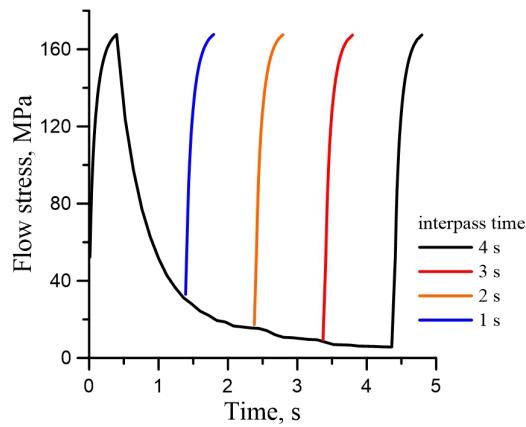


Fig. 4. Results of simulations of the interrupted compression of the cylindrical samples with different interpass times.

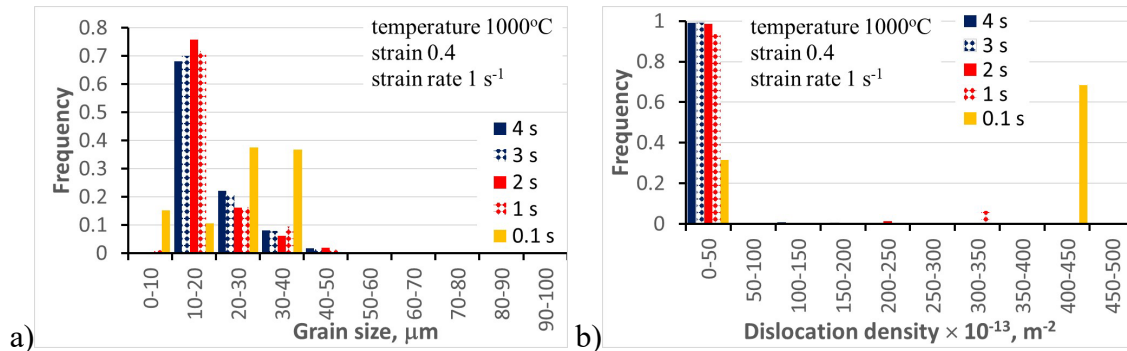


Fig. 5. Histograms of the grain size (a) and dislocation density (b) calculated for various times after the end of deformation.

Varying strain rate test.

In this test the strain rate was changed by an order of magnitude at various stages of the deformation. When external variable models are used, after the change of the conditions the response of the model moves immediately to a new equation of state. On the other hand, it was observed experimentally by many researchers that the response of the material is delayed [16]. This delay is larger for pure metals but it is also observed for alloys (steels) [17]. Below we present the results for the tests when at the strain of 0.4 the strain rate is changed by an order of magnitude (between 0.1 and 1 s<sup>-1</sup>). The test temperature was 1100°C what means that at the strain of 0.4 the dynamic recrystallization has already begun. Changes of the flow stress during varying strain rate tests are shown in Fig. 6a. Delay of the material response is well seen in this figure. Histograms of the dislocation density during these tests are shown in Fig. 6b. Full bars represent dislocation density at the strain 0.4, when the strain rate was changed. The bars with a pattern represent dislocation density at the strain 0.5 (additional strain of 0.1 after the strain rate was changed). It is seen that after this additional strain the state of the system has not reached the state, which was predicted for the constant strain rate.

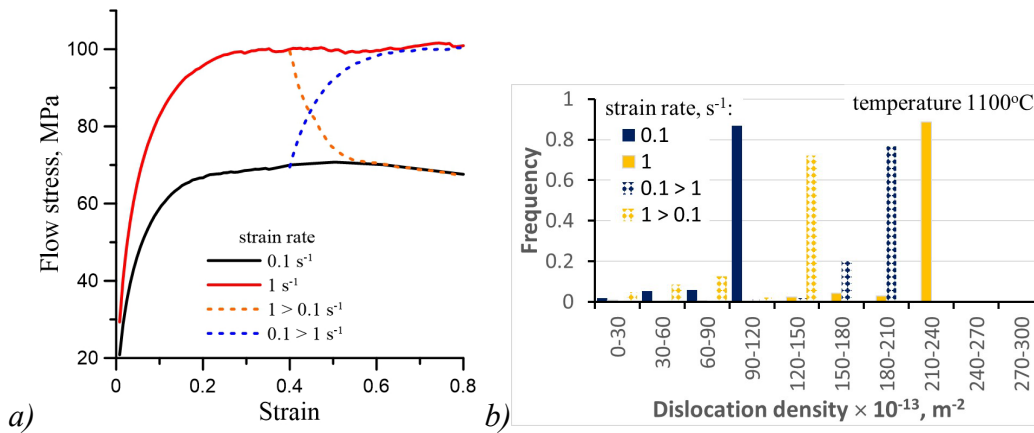


Fig. 6. Changes of the flow stress (a) and histograms of the dislocation density (b) during varying strain rate tests.

### Hot Strip Rolling

The hot strip mill composed of reverse roughing mill, 6-stand continuous finishing mill and 2-section laminar cooling was considered. Rolling of the strip 1500×3 mm was simulated. The results for the finishing mill only are presented below. The work roll radius was 400 mm in all stands and the distance between stands was 5 m. The rolling schedule is given in Table 1, where: *h* - thickness, *r* - reduction, *v* - velocity, *t* - interpass time, *rpm* - rotational velocity of the rolls.

Table 1. Rolling schedule considered in the paper.

pass	<i>h</i> , mm	<i>r</i>	<i>v</i> , m/s	<i>t</i> , s	<i>rpm</i>
0	67	-	0.36	-	-
1	40.6	0.385	0.59	8.5	14.1
2	19.1	0.53	1.26	4	30
3	9.4	0.508	2.56	2	61
4	5.4	0.426	4.44	1.1	106.2
5	3.7	0.315	6.49	0.8	154.9
6	3	0.189	8	0.6	191.1

Two rolling strategies were considered: i) Classical rolling with the end of rolling temperature about 900°C followed by a laminar cooling (V1); ii) Rolling with intensive cooling in finishing mill between 4th and 5th stands as well as between 5th and 6th stands. The end of rolling temperature is below 850°C (V2). Laminar cooling followed and phase transformation model for partially recrystallized material was applied. The objective of V2 was to show the model's capability to predict effect of the austenite deformation on the kinetics of transformations. Calculated time-temperature profiles and load parameters for both variants are shown in Fig. 7. An effect of additional cooling between stands on loads was evaluated, as well, and it was small.

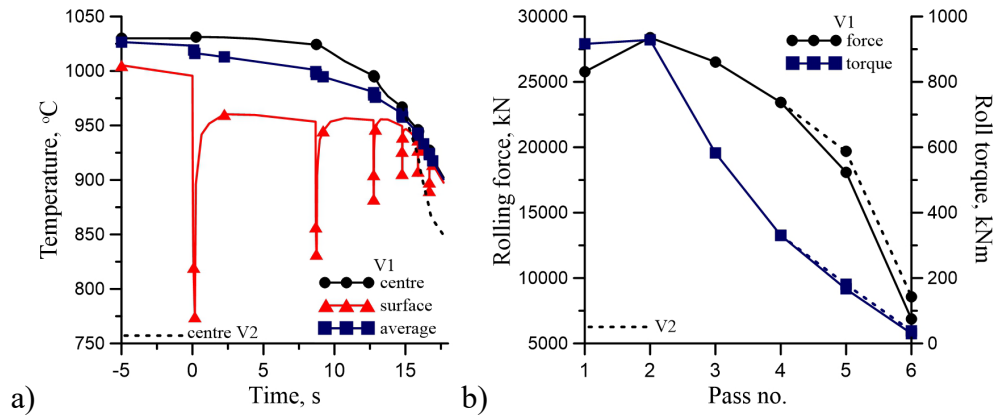


Fig. 7. Calculated time-temperature profiles (a) and load parameters (b) for the two rolling routes.

The stochastic model was applied to calculate distribution of the microstructural parameters during rolling. The selected results for the last 2 passes are shown in Fig. 8. These results show that decrease of the end of rolling temperature leads to an increase of the dislocation density at the beginning of transformations. These results were used as an input data for simulations of the phase transformations during laminar cooling. At this stage the deterministic model of phase transformations described in [12] was applied but the input data were stochastic. The typical laminar cooling system composed of 2 sections and described in [18] was considered. This system allows a 3-stage cooling sequence: fast/slow/fast cooling. In consequence the DP microstructure composed of ferrite and martensite can be obtained. Since the input data for the deterministic phase transformation model were stochastic, calculated phase composition was obtained in the form of histograms, which are shown in Fig. 9a. It is seen in this figure that deformation of the austenite (V2) results in an increase of the ferrite volume fraction. Beyond the ferrite, martensite and small amount of bainite (below 0.02) was predicted by the model.

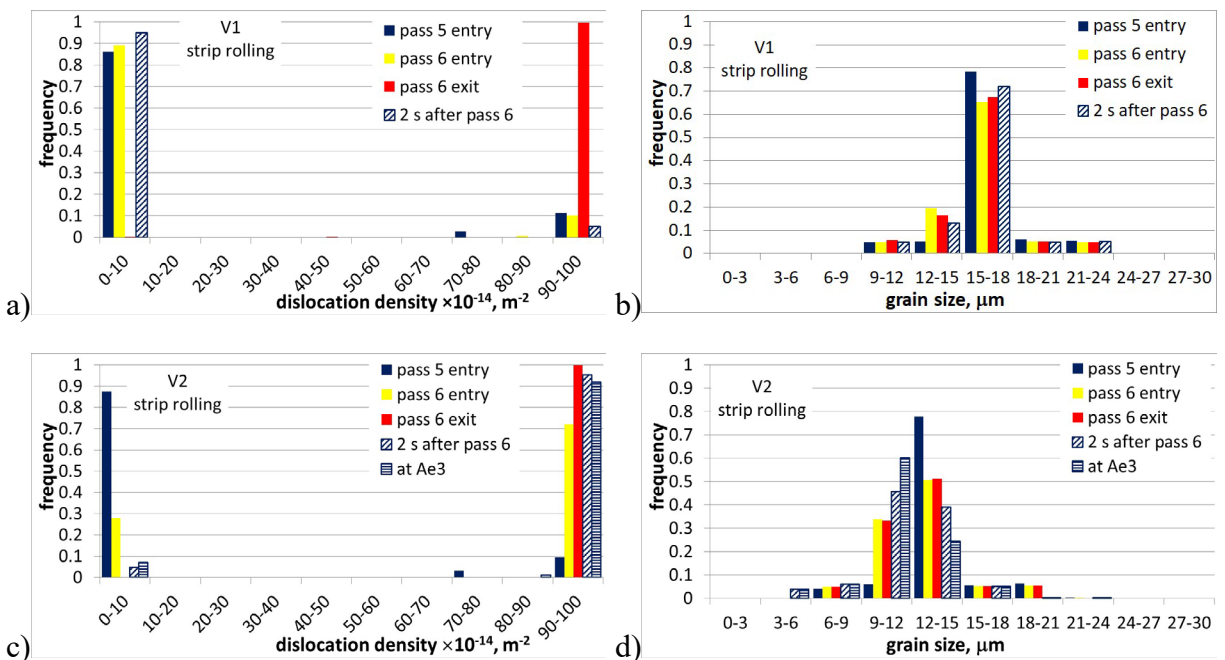


Fig. 8. Calculated histograms of the dislocation density (a,c) and the grain size (b,d) in the last 2 passes of the hot strip rolling according to the rolling schedule V1 (a,b) and V2 (c,d).

Ferrite grain size was calculated by the model, as well. The following deterministic equation proposed in [19] was used:

$$D_{\alpha} = (1 - 0.45\varepsilon_r^{1/2}) \left\{ (-0.4 + 6.37 C_{eq}) + (24.2 - 59 C_{eq}) C_r^{-0.5} + 22 \left[ 1 - \exp(-0.015 D_{\gamma}) \right] \right\} \quad (5)$$

where:  $\varepsilon_r$  - deformation of the austenite at the beginning of transformation,  $C_{eq}$  - carbon equivalent for the steel calculated as  $C_{eq} = C + Mn/6 + (Cr + Mo)/5$ ,  $C_r$  - average cooling rate during ferritic transformation,  $D_{\gamma}$  - austenite grain size prior to transformations in  $\mu m$ .

Deformation of the austenite was zero for the V1 and for the V2 it was evaluated from the dislocation density using reverse KEM model. Although Eq. 5 is deterministic, the input parameters (grain size, dislocation density) are stochastic. In consequence the ferrite grain size was obtained in the form of histograms, which are shown in Fig. 9b. As expected, deformation of the austenite accelerates nucleation of the ferrite and finer grain size was predicted for the rolling schedule V2. Accounting for the random character of the nucleation during phase transformation will be an objective of our future works.

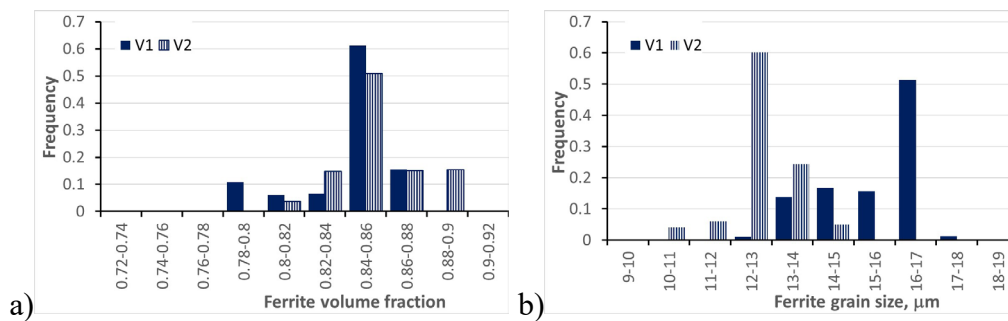


Fig. 9. Calculated histograms of the ferrite volume fraction (a) and the ferrite grain size (b) after laminar cooling of the steel strip.

### Summary

The stochastic model describing evolution of the microstructure during hot strip rolling and laminar cooling is presented in the paper. Numerical tests of the model allowed to draw the following conclusions:

- Identification based on the inverse analysis for the compression tests yielded coefficients in the model, which give good agreement between predictions and measurements of both average values and distributions of various parameters.
- Capability to predict histograms of different microstructural features instead of their average values is the main advantage of the model.
- The model was validated in the interrupted compression test and in the varying strain rate test. Prediction of the model were in qualitative agreement with published information about these tests.
- The model is classified as mean-field model and it does not need explicit representation of the microstructure. In consequence, its computing costs are low.
- The model was applied to simulation of various novel strategies for the industrial hot strip rolling. The results are in agreement with our knowledge about this process, what confirmed model's capability to support a design of the optimal rolling technology.
- The stochastic model for the hot deformation is completed. When this model is coupled with the FE program, it can be applied to any hot forming process. In the phase transformations part it is still the work in progress. In the present work we used deterministic phase



transformation model with stochastic input data (histograms of the dislocation density and the grain size calculated by the hot deformation model). Accounting for the random character of the nucleation during phase transformations will be the objective of future works.

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